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On the Probable Errors of Transit-observing.
By W. H. Finlay, M.A.

In the winter months consecutive transits above and below pole of several stars are often secured on the same night, and a question arises as to how the separate results for azimuthal error are to be combined to form the "adopted" azimuthal error. To settle this question we require to know the probable errors of observation at different declinations, and the following determination of the probable errors of a single wire in transit-observing was undertaken for this purpose.

After my results had been obtained I became aware of the formula given by M. Struve (" Sur l'Emploi de l'Instrument des Passages "), but as the latitude of the Cape is very different from that of Pulkowa I have thought it well to give my results.

The following is the method I have adopted :—

Assuming the adopted wire intervals to be absolutely correct, the difference of the observed interval between two wires from the computed interval gives the actual error made in measuring a distance by observation at two wires; so that by taking the mean of a large number of such differences we get the " average " error of a distance measured by transit over two wires. If M be this "average" error, or mean of the differences without regard to sign, then

$$\text{probable error of a distance} = \text{M} \times [9{\cdot}927046]$$

and

$$\text{probable error of a transit at one wire} = \frac{\text{M}}{\sqrt{2}} \times [9{\cdot}927046].$$

The differences for each star were multiplied by cos (declination) to reduce to the equator, and the stars were grouped in zones according to their North Polar distances. The observations which I have used were made with the transit-circle of the Royal Observatory, Cape of Good Hope, by the three observers who took the chief part in the observations for the Cape Catalogue, 1880; but the period which I found most convenient for my purpose was 1879–1881; the power of the eyepiece used was 180.

The following table gives the probable errors for different N.P.D.'s of an observation at one wire as observed, and also when reduced to their equivalents at the equator :—

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TABLE I.

N.P.D. of Zone.	Mean Z.D. of Zone.	F. p.e. at Obs.	F. No. of Obs.	M. p.e. at Obs.	M. No. of Obs.	P. p.e. at Obs.	P. No. of Obs.	Reduced p.e. F.	Reduced p.e. M.	Reduced p.e. P.
° °	°	s		s		s		s	s	s
80–100	34	0·049	208	0·086	380	0·063	872	0·049	0·086	0·063
100–123	13	0·067	271	0·094	362	0·070	444	0·063	0·088	0·066
124–135	6	0·061	163	0·084	148	0·078	171	0·047	0·064	0·060
135–150	18	0·067	239	0·092	269	0·080	221	0·041	0·057	0·050
150–165	33	0·081	263	0·110	266	0·101	318	0·031	0·042	0·040
166–170	44			0·203	159	0·168	194		0·042	0·035
171–174	48½			0·278	155	0·258	279		0·036	0·034
175–177	52			0·483	340	0·410	626		0·034	0·029
178	54			0·891	120	0·736	115		0·031	0·026
179	55			1·408	88	1·129	125		0·025	0·020
181	57			1·367	60	1·229	97		0·024	0·022
182	58			0·789	26	0·845	110		0·028	0·029
183–185	60			0·553	443	0·475	546		0·039	0·033
186–189	63½			0·260	42	0·302	200		0·034	0·039
190–194	68			0·317	111	0·227	216		0·066	0·047

There were not sufficient observations of circumpolar stars by F in the period 1879–1881 to give a determination of his probable errors beyond 165° N.P.D., and the series for M does not rest on very large numbers of observations except for the zones 175–177 and 183–185. I shall therefore take P as the average observer, and any further conclusions will be based on the figures in the last column of Table I. It will be seen that the probable errors of a single wire by the different observers, though differing in amount, follow the same general law; and that the probable error of observing does not by any means increase in an inverse ratio to the cosine of the star's declination.

If we try to represent our numbers by the formula

$$\text{P.E.}^2 = a^2 + b^2 \sec z \sec^2 \delta,$$

we find

$$a^2 = \cdot003978, \quad b^2 = \cdot000564,$$

and therefore

$$\text{P.E.} = \sqrt{(0^{s}\cdot063)^2 + (0^{s}\cdot024)^2 \sec z \sec^2 \delta}.$$

The representation of the probable errors by this formula, and some variations of it which I have tried, is, however, only moderately satisfactory. The formulæ all failed to give the rapidly diminishing probable errors near the pole. Nor am I surprised at this failure; for, at a distance of 2° or 3° from the pole, the star's disc is in contact with the wire for several seconds,

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and the observer practically alters his method of observing: he fixes his attention on the wire and waits till the star's disc is equally distributed on the two sides of it, and this observation can be made with considerable certainty and accuracy.

To find the error in the determination of the azimuth of the instrument introduced by error in observing, divide these probable errors by sin (zenith distance of group), and we get the following table of probable errors of azimuth reduced from a single wire:—

Table II.

N.P.D. of Star.	p.e. of azimuth.	Combination Weight.	Approx. Weight.
°	s		
168	0·050	2·45	5
172½	0·045	3·16	6
176	0·036	4·57	9
178	0·032	5·57	11
179	0·024	10·00	20
181	0·026		20
182	0·035		11
184	0·038		9
187½	0·044		6
192	0·051		5

so that *four* wires of a star about 178½° N.P.D. are of as much value in determining the azimuthal error as *sixteen* of a star at 170° N.P.D.

We learn another fact from the above table; viz. that at the latitude of the Cape the probable error of the deduced azimuth is the same whether the star be observed above or below the pole; that is to say, the increase of azimuth-factor for the star below pole is counterbalanced by the increased error of observation due to inferior definition at the greater zenith-distance.

The last column in Table II. gives approximate weights for combining azimuthal errors determined from stars at various N.P.D.'s.

Royal Observatory, Cape of Good Hope:
1887, *March* 31.

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Notes on a MS. Eclipse Volume. By the Rev. S. J. Johnson, M.A.

The accompanying MS. volume * gives eclipses in this country for a period of about 2,000 years, from A.D. 538 to A.D. 2500, being recorded ones of both luminaries from the date of the first in 538 to 1200; all solar eclipses visible here from A.D. 1200 to A.D. 2200, omitting a very few in which scarcely a tenth of the Sun's diameter is obscured, including lunar ones for a certain period and large solar eclipses from A.D. 2200 to A.D. 2500.

It will be seen that during the 1000 years from A.D. 1200 to A.D. 2200 there are only eight cases of an interval so long as that through which we are now passing (May 1882 to June 1890) without a solar eclipse visible in England generally.

The only solar eclipses total at Greenwich in the period of 2000 years are those of 878 and 1715, as has been before remarked, unless, perhaps, that of 664 is an instance. Between the eclipse of 1715 and the preceding one that most nearly approached the total phase at Greenwich, viz. that of 1140, there is an interval of 575 years and a month. A similar interval reckoned forwards from 1715 brings us to A.D. 2290, when there is an eclipse soon after sunrise that seems total in the extreme north of the kingdom. A similar interval reckoned previously to 1140 brings us back to February 565, when the Sun appears to have been totally eclipsed across this country just before setting. Though no ancient chroniclers appear to record this directly, yet Matthew of Westminster says, "In this year there were seen many signs in the Sun and Moon."

Melplash Vicarage, Dorset:
April 28.

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* This volume is placed in the Library.